Precalc Warm Up #8-3

Divide $x^3 + 5x^2 + 2x - 8$ by $x + 2$.

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 2 & -8 \\ & & -2 & -6 & 8 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

$x^2 + 3x - 4$

Is $(x+2)$ a factor of $x^3 + 5x^2 + 2x - 8$?

yes, remainder is 0

FACTOR THEOREM

f(x) has a FACTOR (x-k) if and only if $\boxed{f(k) = 0}$

remainder is 0
when you do ÷

Divide $f(x) = x^4 - 3x^3 + 10x^2 - 4x + 8$ by $x - 3$.

$$\begin{array}{r|rrrrr} 3 & 1 & -3 & 10 & -4 & 8 \\ & \downarrow & 3 & 0 & 30 & 78 \\ \hline & 1 & 0 & 10 & 26 & 86 \end{array} \; r$$

Answer: $x^3 + 10x + 26 + \frac{86}{x-3}$

What is the zero of $(x - 3)$? $\boxed{3}$ * Notice k is the zero

Find $f(3) = (3)^4 - 3(3)^3 + 10(3)^2 - 4(3) + 8$

$= 81 - 81 + 90 - 12 + 8$

$= \boxed{86}$

What is the remainder to the original problem? $\boxed{86}$

Cool

REMAINDER THEOREM:

If $f(x)$ is divided by $(x-k)$, then $f(k)$ is the remainder r

when you do syn. ÷

$$\boxed{f(k) = r}$$

HW Questions: p. 201

Divide using long division.

5. $x^4 + 5x^3 + 6x^2 - x - 2 \qquad x + 2$

7. $7x + 3 \qquad x + 2$

$$x+2 \overline{\big)\, 7x + 3}$$

7

$+ \; (-7x + -14)$

-11

$$\boxed{7 + \frac{-11}{x+2}}$$

11. $x^4 + 3x^2 + 1 \qquad x^2 - 2x + 3$

$x^2 + 2x + 4$

$x^2 - 2x + 3 \overline{\big)\, x^4 + 0x^3 + 3x^2 + 0x + 1}$

$-(x^4 - 2x^3 + 3x^2)$

$2x^3 + 0x^2 + 0x + 1$

$-(2x^3 - 4x^2 + 6x)$

$4x^2 - 6x + 1$

$-(4x^2 - 8x + 12)$

$2x - 11$

answer

$$x^2 + 2x + 4 + \frac{2x - 11}{x^2 - 2x + 3}$$

Divide using synthetic division

$\boxed{15.}$ $3x^3 - 17x^2 + 15x - 25$ $\quad x - 5$

$\boxed{19.}$ $-x^3 + 75x - 250$ $\quad x + 10$

$\boxed{23.}$ $10x^4 - 50x^3 - 800$ $\quad x - 6$

$\boxed{27.}$ $-3x^4$ $\quad x - 2$

$\boxed{31.}$ $4x^3 + 16x^2 - 23x - 15$ $\quad x + \frac{1}{2}$

Use synthetic division to show that x is a solution of the third degree polynomial equation, and factor completely.

33. $x^3 - 7x + 6 = 0, \quad x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & \downarrow & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$(x-2)(x^2+2x-3)$

$(x-2)(x+3)(x-1)$

$\boxed{35.}$ $2x^3 - 15x^2 + 27x - 10 = 0, \quad x = \frac{1}{2}$

$$\begin{array}{r|rrrr} & 2 & -15 & 27 & -10 \\ & \downarrow & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$(x-\frac{1}{2})(2x^2-14x+20)$

$(x-\frac{1}{2})\,2\,(x^2-7x+10)$

$(2x-1)$

$(x-5)(x-2)$

$\boxed{37.}$ $x^3 - 3x^2 + 2 = 0, \quad x = 1 + \sqrt{3}$ $\longrightarrow$ $(1-\sqrt{3})$

$$\begin{array}{r|rrrr} 1+\sqrt{3} & 1 & -3 & 0 & 2 \\ & & 1+\sqrt{3} & 1-\sqrt{3} & -2 \\ \hline & 1 & -2+\sqrt{3} & 1-\sqrt{3} & 0 \end{array}$$

$(1+\sqrt{3})(-2+\sqrt{3})$

$-2+\sqrt{3}+(-2\sqrt{3})+3$

$1-\sqrt{3}$

$$\begin{array}{r|rrr} 1-\sqrt{3} & 1 & -2+\sqrt{3} & 1-\sqrt{3} \\ & & 1-\sqrt{3} & -1+\sqrt{3} \\ \hline & 1 & -1 & 0 \end{array}$$

<u>factors</u>

$(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))(x-1)$

$\underbrace{((x-1)-\sqrt{3})((x-1)+\sqrt{3})}(x-1)$

$(x-1)^2-(\sqrt{3})^2$

In Exercises 41–44, express the function in the form $f(x) = (x - k)q(x) + r$ for the given value of k, and demonstrate that $f(k) = r$.

means: divide out the $x-4$ factor and state the result.

means show $f(4)$ = remainder when you did syn division.

41. $f(x) = x^3 - x^2 - 14x + 11, \quad k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \rightarrow r \end{array}$$

$f(x) = (x-4)(x^2+3x-2) + 3$

$f(4) = 4^3 - 4^2 - 14(4) + 11$

$= 48 - 56 + 11$

$= 3$

In Exercises 45–50, use synthetic division to find the required function values.

49. $f(x) = x^3 - 2x^2 - 11x + 52$

(b) $f(-4) = r$ (c) f(1.2)

$$\begin{array}{r|rrrr} -4 & 1 & -2 & -11 & 52 \\ & \downarrow & -4 & 24 & -52 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1.2 & 1 & -2 & -11 & 52 \\ & \downarrow & 1.2 & -.96 & -14.352 \\ \hline & 1 & -.8 & -11.96 & \boxed{37.648} \end{array}$$

$f(1.2) = 37.648$

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$$

Is (x - 5) a likely factor of f(x)?

NO , 18 does not have a factor of 5

Is (3x + 2) a likely factor of f(x)?

NO 3x not a factor of $2x^4$

Is (x - 2) a likely factor of f(x)?

Try it!

$$\begin{array}{r|rrrrr} 2 & 2 & 7 & -4 & -27 & -18 \\ & \downarrow & 4 & 22 & 36 & 18 \\ \hline & 2 & 11 & 18 & 9 & 0 \end{array}$$

yes a factor

no remainder

Factor f(x) completely:

$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

$(x-2)(2x^3+11x^2+18x+9)$ $\pm 1, \cancel{3}, -3, \pm\cancel{9}$

$$\begin{array}{r|rrrr} 3 & 2 & 11 & 18 & 9 \\ & \downarrow & 6 & 34 & \\ \hline & 2 & 17 & \text{NO} & \end{array}$$

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & \downarrow & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$(x-2)(x+3)(2x^2+5x+3)$

$(x-2)(x+3)(2x+3)(x+1)$

Factor $6x^3 - 19x^2 + 16x - 4$ completely.

$$\begin{array}{r|rrrr} 2 & 6 & -19 & 16 & -4 \\ & \downarrow & 12 & -14 & 4 \\ \hline & 6 & -7 & 2 & 0 \end{array}$$

yes x-2 is a factor

No remainder

Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$.

$(x-2)(6x^2-7x+2)$ Is $(x - 2)$ a factor?

$(x-2)(2x-1)(3x-2)$

$f(2) = ?$ 0

Rational Zero Test:

constant

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ has integer coefficients, then every rational zero of $f(x)$ is $\frac{p}{q}$, where p is a factor of constant term a_0 and q is a factor of the leading coefficient a_n.

$$\boxed{\text{Possible Rational Zeros} = \frac{\text{factors of the constant.}}{\text{factors of the lead coeff.}}}$$

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Ex: If $f(x) = 2x^3 + 4x - 5$, → $1, -1, 5, -5, \frac{1}{2}, -\frac{1}{2}, \frac{5}{2}, -\frac{5}{2}$

then $(3x+7)$ isn't a good factor guess.

Which means $\frac{-7}{3}$ is not going to be a zero.

What would be better factor guesses?

Any of the possible rational zeros

Solve by factoring completely:

$x^4 - 3x^3 + x^2 + 3x - 2 = 0$

possible zeros

$$\frac{1 \quad 2 \quad {}^{-}1 \quad {}^{-}2}{1 \quad {}^{-}1}$$

$$\boxed{1, 2, {}^{-}1, {}^{-}2}$$

Not necessary to list duplicates

List all of the possible rational zeros of f, describe end behavior, find all of the zeros and graph

$f(x) = -3x^3 + 20x^2 - 36x + 16$

possible zeros $\dfrac{\pm 4, \pm 16, \pm 2, \pm 1, \pm 8}{\pm 1, \pm 3}$

$4, -4, \cancel{16}, \cancel{-16}, 2, -2, \cancel{1}, \cancel{-1}$

$8, -8, \frac{4}{3}, -\frac{4}{3}, \frac{16}{3}, -\frac{16}{3}, \frac{2}{3}, -\frac{2}{3}$

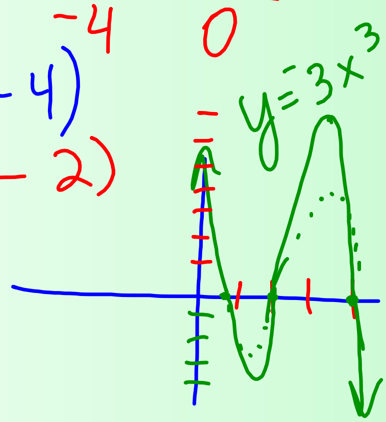
$\frac{1}{3}, -\frac{1}{3}, \frac{8}{3}, -\frac{8}{3}$

$$\begin{array}{r|rrrr} 4 & -3 & 20 & -36 & 16 \\ & \downarrow & -12 & 32 & -16 \\ \hline & -3 & 8 & -4 & 0 \end{array}$$

$(x-4)(-3x^2+8x-4)$

$(x-4)(-3x+2)(x-2)$

$x = 4, \frac{2}{3}, 2$



Evaluate $f\left(\frac{3}{2}\right)$

$f(3)$

$$\begin{array}{r|rrrr} 3 & -3 & 20 & -36 & 16 \\ & \downarrow & -9 & 33 & -9 \\ \hline & -3 & 11 & -3 & 7 \end{array}$$

$+\frac{33}{2}$ $\quad \frac{144}{4}$

$$\begin{array}{r|rrrr} \frac{3}{2} & -3 & +20 & -36 & 16 \\ & \downarrow & -\frac{9}{2} & \frac{93}{4} & \\ \hline & -3 & \frac{31}{2} & -\frac{51}{4} & \end{array}$$

HW: PC book

p. 211 #13, 15, 21 - 51□,

and 53